Extension of the Huttner-Barnett model to a magnetodielectric medium

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Abstract

The Huttner−Barnett model is extended to a magnetodielectric medium by adding a new matter field to this model. The eigenoperators for the coupled system are calculated and electromagnetic field is written in terms of these operators. The electric and magnetic susceptibility of the medium are explicitly derived and shown to satisfy the Kramers−Kronig relations. It is shown that the results obtained in this model are equivalent to the results obtained from the phenomenological methods. .

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1 Introduction

Electromagnetic field (EM) is traditionally quantized by associating a quantum-mechanical harmonic oscillator with each mode of the radiation field in free space [1, 2]. This is achieved by introducing a general position and momentum and expressing the modes in terms of them. The transition from classical to quantum mechanical description takes place by treating the generalized position and momentum quantum mechanically and demanding equal-time

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commutation relations (ETCR) between them. This scheme is called canonical quantization and is based on a Lagrangian density.

In the presence of a linear polarizable medium canonical quantization is retained if the Hattner-Barnett model (HB) is used [3]. This model is based on the Hopfield model [4] and the matter is modeled by a harmonic field which represents the medium excitations. The inclusion of losses into the system can be done by inserting a reservoir consisting of a continuum of harmonic oscillators [5, 6]. Although this model is based on a microscopic model of matter, any microscopic parameter like coupling function between matter and EM field is not appeared in final results and in fact the results are written in terms of the electric susceptibility χ_e which is a macroscopic parameter of the medium.

Since the Hopfield model is written for a homogeneous polarizable matter and not for a magnetizable one, and since the HB model is based on this model, so we should extend the Hopfield model to a magnetizable medium. As mentioned, the final results of HB model do not depend on microscopic parameters, so we can change the microscopic relations such that we get the correct Maxwell equations in the magnetizable medium.

Although by doing so, we may lose the physical interpretation of microscopic interaction, but at least we have a Lagrangian density from which correct Maxwell equations are obtained and a canonical quantization scheme is introduced. In addition, this model has a clear assumption about matter which can not be found in other macroscopic methods.

The common method for the EM field quantization in the presence of matter is the macroscopic method (phenomenological method). The macroscopic approach to EM field quantization begins with Maxwell equations and the loss is modeled by the Langevin force in the form of the noise current operators. A correlation function proportional to the imaginary part of the dielectric function is assumed for the later operations. In this way, a straightforward calculation lead to the field expression in terms of the noise operators and the Green function of Maxwell equations [7]−[11]. In this method ETCR between the EM field operators are postulated and the verification of them justifies the validity of this approach. This model is recently extended to a magnetodielectric medium by adding a new noise operator for magnetization and assuming a correlation between them which is proportional to the imaginary part of magnetic susceptibility [12]. In the conclusion of this paper, we will make a comparison between the results of our model and the macroscopic quantization model.

The paper is arranged as follows: In Sec. 2, the basic theory of electrodynamics in a magnetodielectric medium is reviewed. In sec. 3, we extend the HB model to a magnetizable medium which is a special case of a magnetodielectric medium, in fact, this section is an introduction to the more generalized case of a magnetodielectric medium. In sec. 4, we obtain a Lagrangian density in a magnetodielectric medium from which we quantize the EM field. Finally, the main points are summarized in Sec. 5.

2 Basic equations

The classical Maxwell equations in a magnetodielectric medium are

$$\nabla \times \mathbf{E}(\mathbf{r},t) + \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = 0, \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) - \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} = 0. \tag{2}$$

The constitutive relations relate $\mathbf{D}$ and $\mathbf{H}$ to $\mathbf{E}$ and $\mathbf{B}$. For isotropic linear medium, the relation between polarization and electric field is

$$\mathbf{P}(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{t} dt' \chi_e(\mathbf{r}, t-t')\, \mathbf{E}(\mathbf{r},t')\, dt', \tag{3}$$

where ε_0 is the permittivity of free space. The magnetization and the magnetic induction field are related via a similar expression,

$$\mathbf{M}(\mathbf{r},t) = \kappa_0 \int_{-\infty}^{t} dt' \chi_m(\mathbf{r}, t-t')\, \mathbf{B}(\mathbf{r},t')\, dt', \tag{4}$$

where κ_0 is the inverse permeability of free space. The electric and magnetic susceptibilities are defined in the frequency domain as

$$\chi_\sigma(\mathbf{r},\omega) = \int_{0}^{+\infty} d\tau\, \chi_\sigma(\mathbf{r},\tau)\, e^{\imath\omega\tau}, \tag{5}$$

where the index σ could be e or m. The electric displacement $\mathbf{D}$ is expressed in terms of the electric field $\mathbf{E}$ and the polarization field $\mathbf{P}$ as

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t). \tag{6}$$

The constitutive relation which defines the magnetic field $\mathbf{H}$ in terms of the magnetic induction $\mathbf{B}$ and the magnetization $\mathbf{M}$ is

$$\mathbf{H}(\mathbf{r},t) = \kappa_0 \mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t). \tag{7}$$

The real and imaginary parts of the electric and magnetic susceptibilities should satisfy Kramers-Kronig relations. This shows that the dissipative nature of medium is an immediate consequence of its dispersive character and vice versa. Therefore, in general we are dealing with a dissipative problem. For investigating a dissipative quantum system we should include a reservoir in the process of quantization. After the quantization, the definition of polarization $\mathbf{P}$, and magnetization $\mathbf{M}$, in terms of the Fourier components should be reformed as

$$\hat{\mathbf{P}}(\mathbf{r},t) = \varepsilon_0 \int_0^\infty d\omega\{[\chi_e(\mathbf{r},\omega)\hat{\mathbf{E}}(\mathbf{r},\omega) + \hat{\mathbf{P}}_N(\mathbf{r},\omega)]e^{\imath\omega t} + H.c.\}, \tag{8}$$

and

$$\hat{\mathbf{M}}(\mathbf{r},t) = \kappa_0 \int_0^\infty d\omega\{[\chi_m(\mathbf{r},\omega)\hat{\mathbf{B}}(\mathbf{r},\omega) + \hat{\mathbf{M}}_N(\mathbf{r},\omega)]e^{\imath\omega t} + H.c.\}, \tag{9}$$

respectively, where $\hat{\mathbf{P}}_N(\mathbf{r},\omega)$ and $\hat{\mathbf{M}}_N(\mathbf{r},\omega)$ are polarization and magnetization noise operators and satisfy the fluctuation-dissipation theorem [13].

3 Extension of the Huttner-Barnet model to a magnetizable medium

Here, knowing the Lagrangian density of HB model, we change the form of the interaction term and find a Lagrangian density in a magnetizable medium. Let us write the Lagrangian as

$$\mathcal{L} = \mathcal{L}_{em} + \mathcal{L}_{mat} + \mathcal{L}_{res} + \mathcal{L}_{int}, \tag{10}$$

where

$$\mathcal{L}_{em} = \frac{\varepsilon_0}{2}\mathbf{E}^2 - \frac{\kappa_0}{2}\mathbf{B}^2, \tag{11}$$

is the EM part which can be expressed in terms of vector potential $\mathbf{A}$ and scaler potential U ($\mathbf{E} = -\dot{\mathbf{A}} - \nabla U$ and $\mathbf{B} = \nabla \times \mathbf{A}$).

$$\mathcal{L}_{mat} = \frac{\rho}{2}\dot{\mathbf{X}}^2 - \frac{\rho\omega_0^2}{2}\mathbf{X}^2, \tag{12}$$

is the matter part, modeled by a harmonic oscillator field $\mathbf{X}$ of frequency ω_0.

$$\mathcal{L}_{res} = \int_0^\infty d\omega \left(\frac{\rho}{2}\dot{\mathbf{Y}}_\omega^2 - \frac{\rho\omega^2}{2}\mathbf{Y}_\omega^2 \right) \tag{13}$$

is the reservoir part, consisting of a continuum of harmonic oscillators and

$$\mathcal{L}_{int} = \alpha\nabla \times \mathbf{A} \cdot \mathbf{X} - \int_0^\infty d\omega\, \nu(\omega)\, \mathbf{X} \cdot \dot{\mathbf{Y}}_\omega, \tag{14}$$

is the interaction part which includes the interaction between the light and the magnetization field, with coupling constant α, and also the interaction between the magnetization field and the reservoir oscillator field with the frequency dependent coupling constant $\nu(\omega)$. In this equation the interaction term between the EM and matter field is chosen such that it lead to correct Maxwell equations in a magnetizable medium.

We make two assumptions about $\nu(\omega)$: (i) the analytic continuation of $\mid \nu(\omega) \mid^2$ to negative frequency is an even function and (ii) $\nu(\omega) \neq 0$ for all nonzero frequencies. The first assumption is needed in order to extend the frequency integrals to the negative real axis, while the second one ensure that all the reservoir oscillators couple to the system.

For a purely magnetizable medium $\chi_e = 0$, and we can make $U = 0$ by choosing the Coulomb gauge ($\nabla \cdot \mathbf{A} = 0$). Let us define $\mathbf{M} = \alpha\mathbf{X}$ and write $\mathbf{H}$ as

$$\mathbf{H} = \kappa_0\mathbf{B} - \mathbf{M}. \tag{15}$$

Later we prove that $\mathbf{H}$ in Eq.(15) is in fact the same $\mathbf{H}$ which appears in Maxwell equations.

Now we go to reciprocal space and write all the fields in terms of their spatial Fourier transforms. For example electric field is written as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\,\mathbf{k}\, \tilde{\mathbf{E}}(\mathbf{k}, t)\, e^{\imath\mathbf{k}\cdot\mathbf{r}} \tag{16}$$

(tilde distinguishes the field in real and reciprocal spaces). The total Lagrangian in reciprocal space is

$$L = \int' d^3\mathbf{k}\, (\tilde{\mathcal{L}}_{em} + \tilde{\mathcal{L}}_{mat} + \tilde{\mathcal{L}}_{res} + \tilde{\mathcal{L}}_{int}). \tag{17}$$

Since $\tilde{\mathbf{F}}^*(\mathbf{k},t) = \tilde{\mathbf{F}}(-\mathbf{k},t)$ is satisfied for any arbitrary real field we should restrict the integration to half of the reciprocal space, which is indicated by a prim over the integral in (17). The Lagrangian densities in the reciprocal space are obtained as

$$\tilde{\mathcal{L}}_{em} = \varepsilon_0(\tilde{\mathbf{E}}^2 - c^2\tilde{\mathbf{B}}^2), \tag{18}$$

$$\tilde{\mathcal{L}}_{mat} = \rho\dot{\tilde{\mathbf{X}}}^2 - \rho\omega_0^2\tilde{\mathbf{X}}^2, \tag{19}$$

$$\tilde{\mathcal{L}}_{res} = \int_0^\infty d\omega\,(\rho\dot{\tilde{\mathbf{Y}}}_\omega^2 - \rho\omega^2\tilde{\mathbf{Y}}_\omega^2), \tag{20}$$

$$\tilde{\mathcal{L}}_{int} = \alpha\mathbf{k}\times\tilde{\mathbf{A}}^*\cdot\tilde{\mathbf{X}} - \int_0^\infty d\omega\nu(\omega)\tilde{\mathbf{X}}\cdot\dot{\tilde{\mathbf{Y}}}_\omega^* + c.c.. \tag{21}$$

The Coulomb gauge in reciprocal space can be written as $\mathbf{k}\cdot\tilde{\mathbf{A}}(\mathbf{k},t) = 0$. Using the definition of longitudinal part of matter field, that is $\mathbf{k}\times\tilde{\mathbf{X}}^\parallel(\mathbf{k},t) = 0$, and the coulomb gauge condition, it is easy to prove that $\mathbf{k}\times\tilde{\mathbf{A}}(\mathbf{k},t)\cdot\tilde{\mathbf{X}}^\parallel(\mathbf{k},t) = 0$. This shows that there is no interaction between the longitudinal part of the matter field and EM field. Therefor without losing any generality we can consider matter and reservoir fields as transverse fields.

We introduce unit polarization vectors $\mathbf{e}_\lambda(\mathbf{k})$, $\lambda = 1,2$, which are orthogonal to $\hat{\mathbf{k}}$ and to each other, and decompose the transverse fields along them to get

$$\tilde{\mathbf{A}}(\mathbf{k},t) = \sum_{\lambda=1,2}\tilde{A}_\lambda(\mathbf{k},t)\,\mathbf{e}_\lambda(\mathbf{k}) \tag{22}$$

with similar expressions for the other fields. $\tilde{\mathcal{L}}$ can be used to obtain the conjugate components of the fields

$$-\varepsilon_0\tilde{E}_\lambda = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{A}}_\lambda^*} = \varepsilon_0\dot{\tilde{A}}_\lambda, \tag{23}$$

$$\tilde{P}_\lambda = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{X}}_\lambda^*} = \rho\dot{\tilde{X}}_\lambda, \tag{24}$$

$$\tilde{Q}_{\omega\lambda} = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{Y}}_{\omega\lambda}^*} = \rho\dot{\tilde{Y}}_{\omega\lambda} - v(\omega)\tilde{Y}_{\omega\lambda}. \tag{25}$$

Using the Lagrangian (17) and the expressions for the conjugate variables (23)-(25), we obtain the Hamiltonian as

$$H = \int' d^3\mathbf{k}(\tilde{\mathcal{H}}_{em} + \tilde{\mathcal{H}}_{mat} + \tilde{\mathcal{H}}_{int}), \tag{26}$$

where

$$\tilde{\mathcal{H}}_{em} = \varepsilon_0(\tilde{\mathbf{E}})^2 + \varepsilon_0 c^2 \mathbf{k}^2 \tilde{\mathbf{A}}^2. \tag{27}$$

and

$$\begin{aligned} \tilde{\mathcal{H}}_{mat} &= \frac{\tilde{\mathbf{P}}^2}{\rho} + \rho\tilde{\omega}_0^2\tilde{\mathbf{X}}^2 + \int_0^\infty d\omega(\frac{\tilde{\mathbf{Q}}_\omega^{\ 2}}{\rho} + \rho\omega^2\tilde{\mathbf{Y}}_\omega^2) \\ &+ \int_0^\infty d\omega \frac{(\nu(\omega)}{\rho}\tilde{\mathbf{X}}^* \cdot \tilde{\mathbf{Q}}_\omega) + c.c., \end{aligned} \tag{28}$$

is the matter part, including the interaction between the magnetization and the reservoir. $\tilde{\omega}_0^2 \equiv \omega_0^2 + \int_0^\infty d\omega \frac{v(\omega)^2}{\rho^2}$ is the renormalized frequency of the polarization field and

$$H_{int} = -\alpha \int d^3\mathbf{k}[\mathbf{k} \times \tilde{\mathbf{A}}^* \cdot \tilde{\mathbf{X}} + c.c.], \tag{29}$$

is the interaction between the EM field and the magnetization. The fields are quantized in a standard fashion by demanding ETCR between the variables and their conjugates. For EM field components we have

$$[\hat{\tilde{A}}_\lambda(\mathbf{k}, t), \hat{\tilde{E}}^*_{\lambda'}(\mathbf{k}', t)] = \frac{-\imath\hbar}{\varepsilon_0}\,\delta_{\lambda\lambda'}\,\delta(\mathbf{k} - \mathbf{k}'), \tag{30}$$

and for the matter fields

$$[\hat{\tilde{X}}_\lambda(\mathbf{k}, t), \hat{\tilde{P}}^*_{\lambda'}(\mathbf{k}', t] = \imath\hbar\,\delta_{\lambda,\lambda'}\,\delta(\mathbf{k} - \mathbf{k}'), \tag{31}$$

$$[\hat{\tilde{Y}}_{\omega\lambda}(\mathbf{k}, t), \hat{\tilde{Q}}^*_{\omega'\lambda'}(\mathbf{k}', t)] = \imath\hbar\,\delta_{\lambda\lambda'}\,\delta(\mathbf{k} - \mathbf{k}')\,\delta(\omega - \omega'), \tag{32}$$

with all other equal-time commutators being zero.

Using the inverse Fourier transform, the obtained Hamiltonian in (26) could be written in coordinate space as

$$\begin{aligned}\hat{H} &= \int d^3\mathbf{r}[\frac{\varepsilon_0\hat{\mathbf{E}}^2(\mathbf{r},t)}{2} + \frac{(\kappa_0\nabla\times\hat{\mathbf{A}}(\mathbf{r},t))^2}{2} - \nabla\times\hat{\mathbf{A}}(\mathbf{r},t)\cdot\hat{\mathbf{M}}(\mathbf{r},t)] \\ &+ \hat{H}_{mat}.\end{aligned} \tag{33}$$

The commutation relation between $\hat{\mathbf{E}}(\mathbf{r},t)$ and $\hat{\mathbf{A}}(\mathbf{r},t)$ from, (30), is obtained as

$$[\hat{\mathbf{A}}_\lambda(\mathbf{r},t),\hat{\mathbf{E}}_{\lambda'}(\mathbf{r}',t)] = \imath\frac{\hbar}{\varepsilon_0}\delta^{\perp}_{\lambda\lambda'}(\mathbf{r}-\mathbf{r}'), \tag{34}$$

where $\delta^{\perp}_{\lambda\lambda'}(\mathbf{r}-\mathbf{r}')$ is the transverse delta function [14]. Using the commutation relations (34) and the Hamiltonian (33), we find the Heisenberg equations for $\mathbf{A}(\mathbf{r},\mathbf{t})$ and $\mathbf{E}(\mathbf{r},\mathbf{t})$ as

$$\hat{\mathbf{E}}(\mathbf{r},t) = \frac{\partial\hat{\mathbf{A}}(\mathbf{r},t)}{\partial t}, \tag{35}$$

and

$$\varepsilon_0\frac{\partial\hat{\mathbf{E}}(\mathbf{r},t)}{\partial t} = \kappa_0\nabla\times\nabla\times\hat{\mathbf{A}}(\mathbf{r},t) - \nabla\times\hat{\mathbf{M}}(\mathbf{r},t). \tag{36}$$

Eq.(35) is one of the Maxwell equations and Eq.(36) is also the Maxwell equation in a magnetizable medium if we accept relation (15) and interpret $\hat{\mathbf{M}}(\mathbf{r},t)$ as the magnetization. This shows that the assumed interaction between EM field and matter field lead to the proper Maxwell equations in a magnetic medium.

In the following instead of solving the Heisenberg equation for EM field we write the EM field in terms of eigenoperators of the Hamiltonian (26).

To facilitate the calculations, we introduce a set of three annihilation operators as

$$\hat{a}_\lambda(\mathbf{k},t) = \sqrt{\frac{\epsilon_0}{2\hbar kc}}[kc\hat{\tilde{A}}_\lambda(\mathbf{k},t) - \imath\hat{\tilde{E}}_\lambda(\mathbf{k},t)], \tag{37}$$

$$\hat{b}_\lambda(\mathbf{k},t) = \sqrt{\frac{\rho}{2\hbar\tilde{\omega}_0}}[\tilde{\omega}_0\hat{\tilde{X}}_\lambda(\mathbf{k},t) + \frac{\imath}{\rho}\hat{\tilde{P}}_\lambda(\mathbf{k},t)], \tag{38}$$

$$\hat{b}_\lambda(\mathbf{k},\omega,t) = \sqrt{\frac{\rho}{2\hbar\omega}}[-\imath\omega\hat{\tilde{Y}}_{\omega\lambda}(\mathbf{k},t) + \frac{1}{\rho}\hat{\tilde{Q}}^*_{\omega\lambda}(\mathbf{k},t)]. \tag{39}$$

From the ETCR for the fields, (30)-(32), we obtain the ETCR for above operators as

$$[\hat{a}_\lambda(\mathbf{k},t),\hat{a}^\dagger_\lambda(\mathbf{k}',t)]=\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}'), \tag{40}$$

$$[\hat{b}_\lambda(\mathbf{k},t),\hat{b}^\dagger_\lambda(\mathbf{k}',t)]=\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}'), \tag{41}$$

$$[\hat{b}_\lambda(\mathbf{k},\omega,t),\hat{b}^\dagger_{\lambda'}(\mathbf{k}',\omega',t)]=\delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'). \tag{42}$$

We emphasize that, in contrast to the previous ETCR between conjugate fields, which where correct only in half space, Eqs.(40)-(42) are valid in the hole reciprocal space. By inverting (37)-(39) to find the field operators, and inserting these fields into the Hamiltonian(26), we obtain after integration

$$\hat{H}_{em}=\int d^3\mathbf{k}\sum_{\lambda=1,2}\hbar\omega_{\mathbf{k}}\,\hat{a}^\dagger_\lambda(\mathbf{k})\,\hat{a}_\lambda(\mathbf{k}), \tag{43}$$

$$\begin{aligned}\hat{H}_{mat} &= \int d^3\mathbf{k}\sum_{\lambda=1,2}\{\hbar\tilde{\omega}_0\,\hat{b}^\dagger_\lambda(\mathbf{k})\hat{b}_\lambda(\mathbf{k})+\int_o^\infty \omega\hbar\omega\,\hat{b}^\dagger_\lambda(\mathbf{k},\omega)\,\hat{b}_\lambda(\mathbf{k},\omega)\\ &+ \frac{\hbar}{2}\int_0^\infty d\omega\,V(\omega)\,[\hat{b}^\dagger_\lambda(-\mathbf{k})+\hat{b}_\lambda(\mathbf{k})][\hat{b}^\dagger_\lambda(-\mathbf{k},\omega)+\hat{b}_\lambda(\mathbf{k},\omega)]\},\end{aligned} \tag{44}$$

$$\hat{H}_{int}=\int d^3\mathbf{k}\sum_{\lambda,\lambda'=1,2}\alpha\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\varepsilon_0}}(\hat{a}_\lambda(\mathbf{k})+\hat{a}^\dagger_\lambda(-\mathbf{k}))\sqrt{\frac{\hbar}{2\rho\tilde{\omega}_0}}(\hat{b}_{\lambda'}(\mathbf{k})+\hat{b}^\dagger_{\lambda'}(-\mathbf{k}))\epsilon_{\lambda\lambda'}, \tag{45}$$

where $\epsilon_{\lambda\lambda'}$ is the antisymmetric symbol, $\omega_{\mathbf{k}}=c|\mathbf{k}|$ and $V(\omega)=[\frac{\nu(\omega)}{\rho}]\sqrt{\frac{\omega}{\tilde{\omega}}}$.

The polarization and reservoir parts of the Hamiltonian $\hat{H}_{mat}$ (45) can be diagonalized by using the Fano technique [16] to get a dressed matter field. The diagonalized expression for $\hat{H}_{mat}$ is (the detail of this technique can be found in [5] and we only give the results)

$$\hat{H}_{mat}=\int_0^\infty d\omega\int d^3\mathbf{k}\sum_{\lambda=1,2}\hbar\omega\,\hat{B}^\dagger_\lambda(\mathbf{k},\omega)\,\hat{B}_\lambda(\mathbf{k},\omega), \tag{46}$$

where $\hat{B}^\dagger(\mathbf{k},\omega)$ and $\hat{B}(\mathbf{k},\omega)$ are creation and annihilation operators of the dressed matter field respectively, which satisfy the usual ETCR

$$[\hat{B}_\lambda(\mathbf{k},\omega), \hat{B}'_\lambda(\mathbf{k}',\omega')] = \delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'). \tag{47}$$

They can be expressed in terms of the initial creation and and annihilation operators as

$$\hat{B}(\mathbf{k},\omega) = \alpha_0(\omega)b(\mathbf{k}) + \beta_0(\omega)b^\dagger(\mathbf{k}) + \int_0^\infty d\omega\,\alpha(\omega,\omega')\,b(\mathbf{k},\omega) + \beta(\omega,\omega')\,b^\dagger(\mathbf{k},\omega), \tag{48}$$

and all the coefficient $\alpha_0(\omega)$, $\beta_0(\omega)$, $\alpha_1(\omega,\omega')$ and $\beta_1(\omega,\omega')$ can be obtained in terms of microscopic parameters.

Using the commutators of $\hat{b}$ with $\hat{B}$ and $\hat{B}^\dagger$ together with (48), it is easy to show that

$$\hat{b}_\lambda(\mathbf{k}) = \int_0^\infty d\omega[\alpha_0^*(\omega)\hat{B}_\lambda^\dagger(\mathbf{k},\omega) - \beta_0(\omega)\hat{B}_\lambda(\mathbf{k}\omega)]. \tag{49}$$

We call this model the damped magnetization model. In this model the explicit form of $\alpha_0(\omega)$ and $\beta_0(\omega)$ in terms of the microscopic parameter such as matter field density, ρ, or the coupling between matter and reservoir, $\nu(\omega)$, is not important and we can only accept that such a function exist.

In terms of the new set of operators the interaction part of the Hamiltonian can be written as

$$H_{int} = -\int d^3\mathbf{k}\int_0^\infty d\omega\sum_{\lambda,\lambda'}\frac{\hbar}{2}\Lambda(k)g(\omega)B_\lambda^\dagger(\mathbf{k},\omega)(\hat{a}_\lambda(\mathbf{k}) + \hat{a}_\lambda^\dagger(-\mathbf{k}))\epsilon_{\lambda\lambda'} \tag{50}$$

where $\Lambda(k) = \sqrt{\frac{\alpha^2\omega_\mathbf{k}}{c^2\varepsilon_0\rho\omega_0}}$ and $g(\omega) = (\alpha_0^*(\omega) - \beta_0^*(\omega))$.

Now we can follow two different methods to obtain the time dependence of EM field. One is writing the Heisenberg equation and solving it by using the Laplace transformation. The second one is using the Fano technique and finding the diagonalized $\hat{H}$ and writing EM field in terms of new operators.

Here we following the second method and write the Hamiltonian as

$$\hat{H} = \int_0^\infty d\omega\int d^3\mathbf{k}\sum_{\lambda=1,2}\hbar\omega\hat{C}_\lambda^\dagger(\mathbf{k},\omega)\hat{C}_\lambda(\mathbf{k},\omega), \tag{51}$$

where

$$
\begin{aligned}
\hat{C}_\lambda(\mathbf{k},\omega) &= \tilde{\alpha}_0(k,\omega)a_\lambda(\mathbf{k}) + \tilde{\beta}_0(k,\omega)a_\lambda^\dagger(\mathbf{k}) \\
&+ \int_0^\infty d\omega' \sum_{\lambda'=1,2} [\tilde{\alpha}(k,\omega,\omega')B_{\lambda'}(\mathbf{k},\omega,\omega') + \tilde{\beta}(k,\omega,\omega')B_{\lambda'}^\dagger(\mathbf{k},\omega')]\epsilon_{\lambda\lambda'}.
\end{aligned} \tag{52}
$$

If we follow the algebra of reference [5] the coefficients in the relation above, (52), can be written as

$$\tilde{\alpha}_0(k,\omega) = (\frac{\omega+\omega_\mathbf{k}}{2})\frac{V(\omega,k)}{\omega^2-\omega_\mathbf{k}^2 z(\omega)}, \tag{53}$$

$$\tilde{\beta}_0(k,\omega) = (\frac{\omega-\omega_\mathbf{k}}{2})\frac{V(\omega,k)}{\omega^2-\omega_\mathbf{k}^2 z(\omega)}, \tag{54}$$

$$\tilde{\alpha}(k,\omega,\omega') = \delta(\omega-\omega') + (\frac{\omega_\mathbf{k}}{2})(\frac{V^*(\omega',k)}{\omega-\omega'-\imath\varepsilon})(\frac{V(\omega,k)}{\omega^2-\omega_\mathbf{k}^2}), \tag{55}$$

$$\tilde{\beta}(k,\omega,\omega') = (\frac{\omega_\mathbf{k}}{2})(\frac{V(\omega',k)}{\omega+\omega'})(\frac{V(\omega,k)}{\omega^2-\omega_\mathbf{k}^2 z(\omega)}). \tag{56}$$

where $V(\omega,k) = \Lambda(k)g(\omega)$ and $z(\omega) = 1 - \frac{1}{2\omega_k}\int_{-\infty}^{+\infty} d\omega' \frac{|V(\omega',\mathbf{k})|^2}{\omega'-\omega+i\epsilon}$ and $\epsilon \to 0^+$.

Following the method used in (49) to derive the matter field operator $\hat{b}$ in terms of the dressed matter operators $\hat{B}$ and $\hat{B}^\dagger$, we invert (52) to write the photon annihilation operators $\hat{a}$ and dressed matter operator $\hat{B}$ in terms of the new operators $\hat{C}$ and $\hat{C}^\dagger$ as

$$\hat{a}_\lambda(\mathbf{k}) = \int_0^\infty d\omega[\tilde{\alpha}_0^*(k,\omega)\hat{C}_\lambda(\mathbf{k},\omega) - \tilde{\beta}_0(k,\omega)\hat{C}_\lambda^\dagger(\mathbf{k},\omega)], \tag{57}$$

$$\hat{B}_\lambda(\mathbf{k},\omega) = \int_0^\infty d\omega'[\tilde{\alpha}^*(k,\omega',\omega)\hat{C}_\lambda(\mathbf{k},\omega) - \tilde{\beta}(k,\omega',\omega)\hat{C}_\lambda^\dagger(\mathbf{k},\omega)]. \tag{58}$$

From Eq.(37), $\hat{\mathbf{A}}(\mathbf{r},t)$ is given by

$$\hat{\mathbf{A}}(\mathbf{r},t) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3\mathbf{k} \sum_{\lambda=1,2}\sqrt{\frac{\hbar}{2\epsilon_0\omega_\mathbf{k}}}[\hat{a}_\lambda(\mathbf{k},t)e^{\imath\mathbf{k}\cdot\mathbf{r}} + H.c.]\mathbf{e}_\lambda(\mathbf{k}), \tag{59}$$

and we can use this expression and relation (57) to obtain

$$\begin{aligned}\hat{\mathbf{A}}(\mathbf{r},t) &= \frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3\mathbf{k}\sum_{\lambda=1,2}\sqrt{\frac{\hbar\omega_{\mathbf{k}}^2}{2\varepsilon_0}}\int_0^\infty d\omega\frac{f(\omega)}{\omega^2-\omega_{\mathbf{k}}^2+\omega_{\mathbf{k}}^2\chi_m(\omega)} \\ &\times \ [\hat{C}_\lambda(\mathbf{k},\omega)e^{\imath(\mathbf{k}\cdot\mathbf{r}-\omega t)}+H.c.]\mathbf{e}_\lambda(\mathbf{k}) \qquad (60)\end{aligned}$$

where $f(\omega)$ is defined as $f(\omega)=\frac{\alpha g(\omega)}{\sqrt{c^2\varepsilon_0\rho\omega_0}}$ and $\chi_m(\omega)$ is defined by

$$\chi_m(\omega)=\frac{1}{2}\int_{-\infty}^{+\infty}d\omega'\frac{|f(\omega')|^2}{\omega'-\omega-\imath\varepsilon}=\frac{1}{2}P\int_{-\infty}^{+\infty}\{\frac{|f(\omega')|^2}{\omega'-\omega}\}d\omega'+\frac{1}{2}\imath\pi|f(\omega)|^2. \qquad (61)$$

Later we will show that $\chi_m(\omega)$ in Eq.(61) is the magnetic susceptibility. It can be seen from (61) that the obtained magnetic susceptibility satisfies the Kramers−Kronig relations. In addition $f(\omega)$ can be written as

$$|f(\omega)|^2=\frac{2Im\chi_m(\omega)}{\pi}. \qquad (62)$$

Let us write (60) as

$$\begin{aligned}\hat{\mathbf{A}}(\mathbf{r},t) &= \imath(\frac{\hbar}{8\pi^4})\int d^3\mathbf{k}\sum_{\lambda=1,2}\int_0^\infty d\omega\frac{\omega_{\mathbf{k}}\sqrt{Im\chi_m(\omega)}}{\omega^2-\omega_{\mathbf{k}}^2(1-\chi_m(\omega))} \\ &\times \ [\hat{C}_\lambda(\mathbf{k},\omega)e^{\imath(\mathbf{k}\cdot\mathbf{r}-\omega t)}-H.c.]\mathbf{e}_\lambda(\mathbf{k}). \qquad (63)\end{aligned}$$

Relation (63) only depends on magnetic susceptibility which is a macroscopic quantity. This result is exactly the same as the result of macroscopic approach in a magnetizable medium [12].

Now we show that $\chi_m(\omega)$ is the Fourier transform of the magnetizability of the medium. For this purpose we write

$$\hat{\mathbf{X}}(\mathbf{r},t)=\sqrt{\frac{\hbar}{2\rho\omega_0}}\int d^3\mathbf{k}\sum_{\lambda=1,2}[\hat{b}_\lambda(\mathbf{k})\,e^{\imath\mathbf{k}\cdot\mathbf{r}}+H.c.]\,\mathbf{e}_\lambda(\mathbf{k}), \qquad (64)$$

and using relations (49) and (58) we easily obtain

$$\begin{aligned}\hat{\tilde{M}}_\lambda(\mathbf{k},\omega) &= \sum_{\lambda'}\{\sqrt{\frac{\hbar}{2\rho\omega_0}}\int_0^{+\infty}d\omega'[g(\omega)\alpha(\mathbf{k},\omega',\omega)+g^*(\omega)\beta(\mathbf{k},\omega',\omega)] \\ &\times \ \hat{C}_{\lambda'}(\mathbf{k},\omega')\epsilon_{\lambda'\lambda}\}. \qquad (65)\end{aligned}$$

Using (60), Eq.(65) can be written in real space as

$$\hat{\mathbf{M}}(\mathbf{r},t)=\int_0^\infty d\omega\{[\kappa_0\chi_m(\omega)\nabla\times\hat{\mathbf{A}}(\mathbf{r},\omega)+\hat{\mathbf{M}}_N(\mathbf{r},\omega)]e^{-\imath\omega t}+H.c.\}. \quad (66)$$

Comparing (66) with constitutive equation in (8), the interpretation of $\chi(\omega)$ as a magnetic susceptibility is confirmed. In addition, the magnetization noise operator in (8) is obtained as

$$\hat{\mathbf{M}}_N(\mathbf{r},\omega)=\int d^3\mathbf{k}\sum_{\lambda'=1,2}\sqrt{2\hbar c^2\varepsilon_0 Im\chi_m(\omega)}\hat{C}_{\lambda'}(\mathbf{k},\omega)\epsilon_{\lambda\lambda'}e^{\imath\mathbf{k}\cdot\mathbf{r}}. \quad (67)$$

Considering (52), (67), and (62), the ETCR between $\hat{\mathbf{M}}(\mathbf{r},\omega)$ and $\hat{\mathbf{M}}^\dagger(\mathbf{r},\omega')$ becomes

$$[\hat{M}_{N\lambda}(\mathbf{r},\omega),\hat{M}^\dagger_{N\lambda}(\mathbf{r}',\omega')]=2\hbar c^2\varepsilon_0 Im\chi_m(\omega)\delta_{\lambda\lambda'}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'). \quad (68)$$

Relation (68) satisfies dissipation−fluctuation theorem and is the same as expression of noise operator in macroscopic method [12]. Therefor the supposed Lagrangian is equivalent to macroscopic approach.

4 Extension of the Huttner-Barnett model to a magnetodielectric medium

In pervious section we showed that the HB model can be extended to a magnetizable matter by changing the interaction between EM and the matter fields. The extension of this model to a magnetodielectric matter can be done by considering the following Lagrangian density

$$\mathcal{L}=\mathcal{L}_{em}+\mathcal{L}_{1mat}+\mathcal{L}_{1res}+\mathcal{L}_{1int}+\mathcal{L}_{2mat}+\mathcal{L}_{2res}+\mathcal{L}_{2int}, \quad (69)$$

where $\mathcal{L}_{em}$ is defined in (11), and

$$\mathcal{L}_{1mat}=\frac{\rho}{2}\dot{\mathbf{X}}_1^2-\frac{\rho\omega_0^2}{2}\mathbf{X}_1^2, \quad (70)$$

$$\mathcal{L}_{2mat}=\frac{\rho}{2}\dot{\mathbf{X}}_2^2-\frac{\rho\omega_0^2}{2}\mathbf{X}_2^2, \quad (71)$$

are the matter parts of Lagrangian, which are modeled by two distinct harmonic oscillator fields X_1 and X_2 with the same frequency ω_0. In the following we call X_1 and X_2 the polarization and magnetization fields respectively. It should be noted that taking the same frequency for both polarization and magnetization fields does not affect the result. The Lagrangians describing the reservoir are defined by

$$\mathcal{L}_{1res} = \int_0^\infty d\omega \left(\frac{\rho}{2} \dot{\mathbf{Y}}_{1\omega}^2 - \frac{\rho\omega^2}{2} \mathbf{Y}_{1\omega}^2 \right), \tag{72}$$

$$\mathcal{L}_{2res} = \int_0^\infty d\omega \left(\frac{\rho}{2} \dot{\mathbf{Y}}_{2\omega}^2 - \frac{\rho\omega^2}{2} \mathbf{Y}_{2\omega}^2 \right), \tag{73}$$

and

$$\mathcal{L}_{1int} = -\alpha_1 \mathbf{A} \cdot \dot{\mathbf{X}}_1 - \int_0^\infty d\omega \nu_1(\omega) \mathbf{X}_1 . \dot{\mathbf{Y}}_{1\omega}, \tag{74}$$

$$\mathcal{L}_{2int} = \alpha_2 \nabla \times \mathbf{A} \cdot \mathbf{X}_2 - \int_0^\infty d\omega \nu_2(\omega) \mathbf{X}_2 . \dot{\mathbf{Y}}_{2\omega}, \tag{75}$$

are the interaction parts. The coupling functions $\nu_1(\omega)$ and $\nu_2(\omega)$, satisfy the same assumptions we assumed for $\nu(\omega)$ in (14).

The displacement and magnetic fields are defined respectively by

$$\mathbf{D}(\mathbf{r}, t) = \varepsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}, (\mathbf{r}, t) \tag{76}$$

and

$$\mathbf{H}(\mathbf{r}, t) = \kappa_0 \mathbf{B}(\mathbf{r}, t) - \mathbf{M}(\mathbf{r}, t), \tag{77}$$

where we have defined $\mathbf{P}(\mathbf{r}, t) \equiv -\alpha_1 \mathbf{X}_1(\mathbf{r}, t)$ and $\mathbf{M}(\mathbf{r}, t) \equiv +\alpha_2 \mathbf{X}_2(\mathbf{r}, t)$.

Again we choose the Coulomb gauge, but since the medium is polarizable the scaler potential does not vanish. Since $\dot{U}$ does not appear in the Lagrangian, U is not a proper dynamical variable and should be written in terms of the proper dynamical variables $\mathbf{A}$, $\mathbf{X}$, and $\mathbf{Y}_\omega$. This can be done by going to reciprocal space and writing the Lagrangian as

$$L = \int' d^3\mathbf{k} (\tilde{\mathcal{L}}_{em} + \sum_{\mathtt{i}=1,2} \tilde{\mathcal{L}}_{\mathtt{i}mat} + \tilde{\mathcal{L}}_{\mathtt{i}res} + \tilde{\mathcal{L}}_{\mathtt{i}int}). \tag{78}$$

The Lagrangian densities in this space are obtained as

$$\tilde{\mathcal{L}}_{em} = \varepsilon_0 \tilde{\mathbf{E}}^2 - \kappa_0 \tilde{\mathbf{B}}^2, \tag{79}$$

$$\tilde{\mathcal{L}}_{\mathtt{i}mat} = \rho\dot{\tilde{\mathbf{X}}}_{\mathtt{i}}^2 - \rho\omega_0^2\tilde{\mathbf{X}}_{\mathtt{i}}^2, \tag{80}$$

$$\tilde{\mathcal{L}}_{\mathtt{i}res} = \int_0^\infty d\omega(\rho\dot{\tilde{\mathbf{Y}}}_{\mathtt{i}\omega}^2 - \rho\omega^2\tilde{\mathbf{Y}}_{\mathtt{i}\omega}^2) \tag{81}$$

with $\mathtt{i} = 1, 2$, and

$$\tilde{\mathcal{L}}_{1int} = [-\alpha_1\tilde{\mathbf{A}}^* \cdot \dot{\tilde{\mathbf{X}}} - \int_0^\infty d\omega\nu(\omega)\tilde{\mathbf{X}} \cdot \dot{\tilde{\mathbf{Y}}}_\omega^*] + c.c., \tag{82}$$

$$\tilde{\mathcal{L}}_{2int} = \alpha_2\mathbf{k} \times \tilde{\mathbf{A}}^* \cdot \tilde{\mathbf{X}}_2 - \int_0^\infty d\omega\nu(\omega)\tilde{\mathbf{X}}_2 \cdot \dot{\tilde{\mathbf{Y}}}_{2\omega}^* + c.c.. \tag{83}$$

Using the Euler-Lagrange equation for $\dot{\tilde{U}}^*$ we find

$$\tilde{U}(\mathbf{k}, t) = \imath\frac{\alpha_1}{\varepsilon_0}(\frac{\mathbf{k} \cdot \tilde{\mathbf{X}}(\mathbf{k}, t)}{\mathbf{k}^2}). \tag{84}$$

Now if we decompose the longitudinal and transverse parts of the fields, then the total lagrangian can be written as the sum of two independent transverse and longitudinal parts as

$$L = L^\parallel + L^\perp, \tag{85}$$

where

$$L^\perp = \int' d^3\mathbf{k}\tilde{\mathcal{L}}_{em}^\perp + \sum_{\mathtt{i}=1,2}(\tilde{\mathcal{L}}_{\mathtt{i}mat}^\perp + \tilde{\mathcal{L}}_{\mathtt{i}res}^\perp + \tilde{\mathcal{L}}_{\mathtt{i}int}^\perp), \tag{86}$$

and

$$\tilde{\mathcal{L}}_{em}^\perp = \varepsilon_0(\dot{\tilde{\mathbf{A}}}^2 - c^2\tilde{\mathbf{B}}^2), \tag{87}$$

$$\tilde{\mathcal{L}}_{\mathtt{i}mat}^\perp = \rho\dot{\tilde{\mathbf{X}}}_{\mathtt{i}}^{\perp 2} - \rho\omega_0^{\perp 2}\tilde{\mathbf{X}}_{\mathtt{i}}^{\perp 2}, \tag{88}$$

$$\tilde{\mathcal{L}}_{\mathtt{i}res}^\perp = \int_0^\infty d\omega(\rho\dot{\tilde{\mathbf{Y}}}_{\mathtt{i}\omega}^2 - \frac{\rho\omega^2}{2}\tilde{\mathbf{Y}}_{\mathtt{i}\omega}^{\perp 2}), \tag{89}$$

$$\tilde{\mathcal{L}}_{1int}^\perp = -\alpha_1\tilde{\mathbf{A}} \cdot \dot{\tilde{\mathbf{X}}}^\perp - \int_0^\infty d\omega\nu_1(\omega)\tilde{\mathbf{X}}^\perp \cdot \dot{\tilde{\mathbf{Y}}}_\omega^{\perp *} + c.c., \tag{90}$$

$$\tilde{\mathcal{L}}_{2int}^\perp = \alpha_2\mathbf{k} \times \tilde{\mathbf{A}} \cdot \tilde{\mathbf{X}}_2^\perp - \int_0^\infty d\omega\nu_2(\omega)\tilde{\mathbf{X}}_2^\perp \cdot \dot{\tilde{\mathbf{Y}}}_{2\omega}^{*\perp} + c.c.. \tag{91}$$

As in the previous section, we put away the longitudinal part of magnetization field. So the longitudinal part of the Lagrangian consists of only the polarization and EM field. Using (84), (79) and (80) the longitudinal part of the Lagrangian in terms of the polarization field can be written as

$$L^{\parallel} = \int' d^3\mathbf{k}\tilde{\mathcal{L}}_1^{\parallel}, \tag{92}$$

where

$$\begin{aligned} \tilde{\mathcal{L}}_1^{\parallel} &= \rho\dot{\tilde{\mathbf{X}}}_1^{\parallel 2} - \rho\omega_L^2\tilde{\mathbf{X}}_1^{\parallel 2} + \int_0^{+\infty} d\omega(\rho\dot{\tilde{\mathbf{Y}}}_{1\omega}^{\parallel 2} - \rho\omega^2\tilde{\mathbf{Y}}_{1\omega}^{\parallel 2}) \\ &- \int_0^{+\infty} d\omega v_1(\omega)(\tilde{\mathbf{X}}_1^{\parallel *}\cdot\dot{\tilde{\mathbf{Y}}}_{1\omega}^{\parallel} + \dot{\tilde{\mathbf{X}}}_1^{\parallel *}\cdot\tilde{\mathbf{Y}}_{1\omega}^{\parallel}). \end{aligned} \tag{93}$$

The ω_L is the longitudinal frequency and is defined by $\omega_L \equiv \sqrt{\omega_0^2 + \omega_c^2}$ where $\omega_c^2 = \frac{\alpha_1^2}{\rho\epsilon_0}$. The link between the transverse and longitudinal parts is given by the total electric field, which is written as

$$\tilde{\mathbf{E}}(\mathbf{k},t) = -\dot{\tilde{\mathbf{A}}}(\mathbf{k},t) + \frac{\alpha_1}{\varepsilon_0}\tilde{\mathbf{X}}^{\parallel}(\mathbf{k},t). \tag{94}$$

Using (94) and the definition of the displacement field $\mathbf{D}$ given by (76), we recover the fact that the displacement vector is a purely transverse field.

In this work, we are mainly interested in the transverse fields and shall only present the detailed quantization of the transverse part of the Lagrangian. In what follows we restrict ourselves to transverse fields and omit the superscript $\perp$. We use the same unit polarization vectors $\mathbf{e}_\lambda(\mathbf{k})$, $\lambda = 1, 2$ an find the the conjugate variables from $\tilde{\mathcal{L}}$ as

$$-\varepsilon_0\tilde{E}_\lambda = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{A}}_\lambda^*} = \varepsilon_0\dot{\tilde{A}}_\lambda, \tag{95}$$

$$\tilde{P}_{1\lambda} = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{X}}_{1\lambda}^*} = \rho\dot{\tilde{X}}_{1\lambda} - \alpha_1\tilde{A}_\lambda, \tag{96}$$

$$\tilde{P}_{2\lambda} = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{X}}_{2\lambda}^*} = \rho\dot{\tilde{X}}_{2\lambda}, \tag{97}$$

$$\tilde{Q}_{\mathtt{i}\omega\lambda} = \frac{\partial\mathcal{L}}{\partial\dot{\tilde{Y}}_{\mathtt{i}\omega\lambda}^*} = \rho\dot{\tilde{Y}}_{\mathtt{i}\omega\lambda} - v_{\mathtt{i}}(\omega)\tilde{Y}_{\mathtt{i}\omega\lambda}. \tag{98}$$

For the particular type of the coupling between light and polarization field (?), the conjugate of $\tilde{\mathbf{A}}$ is the transverse electric field $-\epsilon_0\tilde{\mathbf{E}}$. A canonical transformation leading to a $\tilde{\mathbf{E}}\cdot\tilde{\mathbf{X}}$ type of coupling, gives the displacement field $-\tilde{\mathbf{D}}$ as the conjugate of $\tilde{\mathbf{A}}$. Naturally, these two possibilities lead to the same results. We choose here the first possibility in order to keep as close as possible to the classical theory, where $\mathbf{E}$ is usually considered as the fundamental variable.

Following pervious section we can obtain the Hamiltonian from the Lagrangian (86) and the conjugate variables (95)-(98), as

$$H = \int' d^3\mathbf{k}(\tilde{\mathcal{H}}_{em} + \sum_{\mathtt{i}=1,2}\tilde{\mathcal{H}}_{\mathtt{i}mat} + \tilde{\mathcal{H}}_{\mathtt{i}int}), \tag{99}$$

where

$$\tilde{\mathcal{H}}_{em} = \epsilon_0(\tilde{\mathbf{E}})^2 + \epsilon_0\tilde{\omega}_{\mathbf{k}}^2\tilde{\mathbf{A}}^2, \tag{100}$$

is the electromagnetic energy density and $\tilde{\omega}_{\mathbf{k}}$ is defined by $\tilde{\omega}_{\mathbf{k}}^2 \equiv c^2(k^2+k_c^2)$ with $k_c \equiv \frac{\omega_c}{c} = \sqrt{\frac{\alpha_1^2}{\rho c^2\epsilon_0}}$, the Hamiltonian densities

$$\begin{aligned}\tilde{\mathcal{H}}_{\mathtt{i}mat} &= \frac{\tilde{\mathbf{P}}_{\mathtt{i}}^{2\perp}}{\rho} + \rho\tilde{\omega}_0^2\tilde{\mathbf{X}}_{\mathtt{i}}^{2\perp} + \int_0^\infty d\omega(\frac{\tilde{\mathbf{Q}}_{\mathtt{i}\omega}^{\perp}{}^2}{\rho} + \rho\omega^2\tilde{\mathbf{Y}}_{\mathtt{i}\omega}^{\perp 2}) \\ &+ \int_0^\infty d\omega(\frac{V_{\mathtt{i}}(\omega)}{\rho}\tilde{\mathbf{X}}_{\mathtt{i}}^{\perp *}\cdot\tilde{\mathbf{Q}}_{\mathtt{i}\omega}^{\perp}) + c.c.,\end{aligned} \tag{101}$$

are the energy densities of the matter fields, where $V_{\mathtt{i}}(\omega)$, $(i=1,2)$, are defined in (45), and

$$\tilde{\mathcal{H}}_{1in} = \frac{\alpha_1}{\rho_1}(\tilde{\mathbf{A}}^*\cdot\tilde{\mathbf{P}}_1 + c.c.), \tag{102}$$

$$\tilde{\mathcal{H}}_{2in} = -\frac{\alpha_2}{\rho_2}(\mathbf{k}\times\tilde{\mathbf{A}}^*\cdot\tilde{\mathbf{X}}_2 + c.c.), \tag{103}$$

are the interaction energies between the EM field and the matter fields.

As usual, we demand ETCR between the variables and their conjugates and introduce the annihilation operators as

$$\hat{a}_\lambda(\mathbf{k},t) = \sqrt{\frac{\epsilon_0}{2\hbar\tilde{k}c}}(\tilde{k}c\hat{\tilde{A}}_\lambda(\mathbf{k},t) - \imath\hat{\tilde{E}}_\lambda^*(\mathbf{k},t)), \tag{104}$$

$$\hat{b}_{\mathtt{i}\lambda}(\mathbf{k},t)=\sqrt{\frac{\rho}{2\hbar\tilde{\omega}_0}}(\tilde{\omega}_0\hat{\tilde{X}}_{\mathtt{i}\lambda}(\mathbf{k},t)+\frac{\imath}{\rho}\hat{\tilde{P}}^*_{\mathtt{i}\lambda}(\mathbf{k},t)), \tag{105}$$

$$\hat{b}_{\mathtt{i}\lambda}(\mathbf{k},,\omega,t)=\sqrt{\frac{\rho}{2\hbar\omega}}(-\imath\omega\hat{\tilde{Y}}_{\mathtt{i}\omega\lambda}(\mathbf{k},t)+\frac{1}{\rho}\hat{\tilde{Q}}^*_{\mathtt{i}\omega\lambda}(\mathbf{k},t)), \tag{106}$$

which satisfy the standard bosonic commutation relations.

The Hamiltonian (99) can be written in terms of the annihilation and creation operators defined in Eqs.(107)−(109). Since the structure of the matter fields in the Hamiltonian (101) are the same as H_{mat} in (28), so the Hamiltonian of the matter can be diagonalized using the same method. The total Hamiltonian in terms of eigenoperators of the matter and reservoir fields can be written as

$$\begin{aligned}\hat{H} &= \int d^3\mathbf{k}\{\hbar\tilde{\omega}_{\mathbf{k}}\hat{a}^\dagger_\lambda(\mathbf{k})\hat{a}_\lambda(\mathbf{k})+\int_0^\infty d\omega\hbar\omega\sum_{\mathtt{i}}\hat{B}^\dagger_{\mathtt{i}\lambda}(\mathbf{k},\omega)\hat{B}_{\mathtt{i}\lambda}(\mathbf{k},\omega)\\ &+ \frac{\hbar}{2}\Lambda_1(k)\int_0^\infty d\omega\{g_1(\omega)\hat{B}^\dagger_{1\lambda}(\mathbf{k},\omega)[\hat{a}_\lambda(\mathbf{k})+\hat{a}^\dagger_\lambda(-\mathbf{k})]+H.c.\}\\ &- \frac{\hbar}{2}\Lambda_2(k)\int_0^\infty d\omega\sum_{\lambda'}\{g_2(\omega)\hat{B}^\dagger_{2\lambda'}(\mathbf{k},\omega)[\hat{a}_\lambda(\mathbf{k})+\hat{a}^\dagger_\lambda(-\mathbf{k})]\epsilon_{\lambda\lambda'}+H.c.\}\},\end{aligned} \tag{107}$$

where the annihilation operators of the polarization and magnetization fields are

$$\hat{b}_{\mathtt{i}\lambda}(\mathbf{k},t)=\int_0^\infty d\omega[\alpha_0(\omega)\hat{B}^\dagger_{\mathtt{i}\lambda}(\omega,\mathbf{k},t)-\beta_0(\omega)\hat{B}_{\mathtt{i}\lambda}(\omega,\mathbf{k},t)], \tag{108}$$

where $\Lambda_1(k)\equiv\sqrt{\frac{\tilde{\omega}_0 c k_c^2\alpha_1^2}{\tilde{k}}}$, $\Lambda_2(k)=\sqrt{\frac{\alpha_2^2k^2}{\varepsilon_0\rho\tilde{\omega}_{\mathbf{k}}\tilde{\omega}_0}}$, $g_1(\omega)=\imath(\alpha_{01}(\omega)+\beta_{01}(\omega))$, $g_2(\omega)=(\alpha^*_{02}(\omega)-\beta^*_{02}(\omega))$ and the $\mathbf{k}$ integration has been extended to full reciprocal space.

In Eq.(107), EM field is coupled with two distinct reservoirs and is different from the usual Huttner model which only contains one reservoir, so we represent here the details of the diagonalization process of the Hamiltonian (107).

The diagonalization of $\hat{H}$ can be achieved by introducing the operators $\hat{C}(\mathbf{k},\omega)$ as

$$\begin{aligned}\hat{C}_\lambda(\mathbf{k},\omega) &= \alpha_0(k,\omega)\hat{a}_\lambda(\mathbf{k}) + \beta_0(k,\omega)\hat{a}_\lambda^\dagger(\mathbf{k}) \\ &+ \int_0^\infty d\omega'[\alpha_1(k,\omega,\omega')\hat{B}_1(\mathbf{k},\omega') + \beta_1(k,\omega,\omega')\hat{B}_1^\dagger(\mathbf{k},\omega')] \\ &+ \int_0^\infty d\omega' \sum_{\lambda'}[\alpha_2(k,\omega,\omega')\hat{B}_{2\lambda'}(\mathbf{k},\omega') + \beta_2(k,\omega,\omega')\hat{B}_{2\lambda'}^\dagger(\mathbf{k},\omega')]\epsilon_{\lambda,\lambda'},\end{aligned} \tag{109}$$

where the coefficients are chosen such that the operators $\hat{C}(\mathbf{k},\omega)$ satisfy the eigenoperator equation

$$[\hat{C}(\mathbf{k},\omega),\hat{H}] = \hbar\omega\hat{C}(\mathbf{k},\omega). \tag{110}$$

This equation, together with the expansion of the Hamiltonian (107) and the definition of $\hat{C}(\mathbf{k},\omega)$ in (109), lead to the following linear equations between the coefficients

$$\begin{aligned}\alpha_0(k,\omega)\omega &= \alpha_0(k,\omega)\omega_{\mathbf{k}} \\ &+ \sum_{\mathtt{i}=1,2}\frac{1}{2}\int_0^\infty d\omega'[\alpha_1(k,\omega,\omega')V_{\mathtt{i}}(k,\omega') - \beta_{\mathtt{i}}^*(k,\omega,\omega')V_{\mathtt{i}}(k,\omega')],\end{aligned} \tag{111}$$

$$\begin{aligned}\beta_0(k,\omega)\omega &= -\beta_0(k,\omega)\omega_{\mathbf{k}} \\ &+ \sum_{\mathtt{i}=1,2}\frac{1}{2}\int_0^\infty d\omega'[\alpha_{\mathtt{i}}(k,\omega,\omega')V_{\mathtt{i}}(k,\omega') - \beta_{\mathtt{i}}(\mathbf{k},\omega,\omega')V_{\mathtt{i}}(k,\omega')],\end{aligned} \tag{112}$$

$$\alpha_{\mathtt{i}}(k,\omega,\omega')\omega = \frac{1}{2}[\alpha_0(k,\omega) - \beta_0(k,\omega)]V_{\mathtt{i}}^*(k,\omega') + \alpha_{\mathtt{i}}(k,\omega,\omega')\omega', \tag{113}$$

$$\beta_{\mathtt{i}}(k,\omega,\omega')\omega = \frac{1}{2}[\alpha_0(k,\omega) - \beta_0(k,\omega)]V_{\mathtt{i}}^*(k,\omega') - \beta_{\mathtt{i}}(k,\omega,\omega')\omega', \tag{114}$$

where $V_{\mathtt{i}}(k,\omega) = \Lambda_{\mathtt{i}}(k)g_{\mathtt{i}}(\omega)$.

These set of equations can be easily solved to obtain $\beta_0(k,\omega)$, $\alpha_{\mathtt{i}}(k,\omega,\omega')$ and $\beta_{\mathtt{i}}(k,\omega,\omega')$ in terms of $\alpha_0(\mathbf{k},\omega)$. Subtracting (112) from (111) we obtain

$$\beta_0(k,\omega) = \frac{\omega - \tilde{\omega}_{\mathbf{k}}}{\omega + \tilde{\omega}_{\mathbf{k}}}\alpha_0(k,\omega). \tag{115}$$

We now replace (115) for $\beta_0(\mathbf{k},\omega)$ in (113) and (114), and find

$$\alpha_{\mathtt{i}}(k,\omega,\omega') = [P(\frac{1}{\omega-\omega'}) + y_{\mathtt{i}}(k,\omega)]V_{\mathtt{i}}^*(k,\omega')\frac{\tilde{\omega}_{\mathbf{k}}}{\omega+\tilde{\omega}_{\mathbf{k}}}\alpha_0(\omega), \tag{116}$$

$$\beta_{\mathtt{i}}(k,\omega,\omega') = [\frac{1}{\omega+\omega'}]V_{\mathtt{i}}(k,\omega')\frac{\tilde{\omega}_{\mathbf{k}}}{\omega+\tilde{\omega}_{\mathbf{k}}}\alpha_0(k,\omega), \tag{117}$$

where P means the Cauchy principal value. The relation between functions $y_1(k,\omega)$ and $y_2(k,\omega)$ can be obtained by substituting the expressions for $\alpha_i(k,\omega,\omega')$ and $\beta_i(k,\omega,\omega')$ in (116) and (117) into (111). Using the definitions of $V_1^2(k,\omega)$ and $V_2^2(k,\omega$ in (107) it is easy to show that they are odd functions of frequency ω. We use this fact to extend the integral in the negative frequency region and obtain the relation between functions $y_1(k,\omega)$ and $y_2(k,\omega)$ as

$$\begin{aligned} V_1^2(k,\omega)y_1(k,\omega) + V_2^2(k,\omega)y_2(k,\omega) &= \frac{\omega^2-\tilde{\omega}_{\mathbf{k}}^2}{\tilde{\omega}_{\mathbf{k}}} + \frac{1}{2}P\int_{-\infty}^{+\infty} d\omega' \frac{V_1^2(k,\omega')}{\omega'-\omega} \\ &+ \frac{1}{2}P\int_{-\infty}^{+\infty} d\omega' \frac{V_2^2(k,\omega')}{\omega'-\omega}. \end{aligned} \tag{118}$$

In order to calculate $\alpha_0(k,\omega)$, we impose the standard commutation relation on $\hat{C}(\mathbf{k},\omega)$

$$[\hat{C}(\mathbf{k},\omega), \hat{C}^\dagger(\mathbf{k}',\omega)] = \delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}'). \tag{119}$$

Using the expression for $\hat{C}(\mathbf{k},\omega)$ given by (109) and the set of equations $\alpha_0(k,\omega)$ (115)$-$(117), we can find $\alpha_0(k,\omega)$ (up to a phase factor) in terms of the $y_1(k,\omega)$ and $y_2(k,\omega)$. By taking a suitable phase factor and doing some routine but tedious calculations we find the following expression for $\alpha_0(k,\omega)$

$$\alpha_0(k,\omega) = \frac{\omega+\tilde{\omega}_{\mathbf{k}}}{\tilde{\omega}_k}\{\frac{1}{(y_1(k,\omega)+\imath\pi)^2V_1^2(k,\omega) + (y_2(k,\omega)+\imath\pi)^2V_2^2(k,\omega)}\}^{\frac{1}{2}}. \tag{120}$$

From the Eqs.(109) and (111)$-$(114), we can obtain two independent sets of operators, $\hat{C}$ and $\hat{C}'$ which satisfy the following commutation relation

$$[\hat{C}(k,\omega), \hat{C}'^{\dagger}(k,\omega)] = 0. \tag{121}$$

For obtaining these operators, we first choose $y_1(k,\omega)$ and $y_2(k,\omega)$ such that they satisfy (118) and then using (120), we find $\hat{C}$ and $\hat{C}^{\dagger}$. For obtaining $\hat{C}'$ and $\hat{C}'^{\dagger}$, we should choose y_1' and y_2' such that they satisfy (118) and also the derived operators $\hat{C}'$ and $\hat{C}'^{\dagger}$ from them should satisfy (121). So, for defining y_1' and y_2' there are two equations and accordingly they can be determined uniquely. operator $\hat{C}$.

Since in Eq.(118), $y_1(k,\omega)$ and $y_2(k,\omega)$ can not be determined uniquely, so we have a freedom in determining operators $\hat{C}$, $\hat{C}'$. But we do not lose any generality by taking a special solution since these operators are all equivalent up to a Bogoliubov transformation.

To facilitate the calculations, we choose $y_1(k,\omega)$ and $y_2(k,\omega)$ such that they satisfy in (118) and the following relation

$$V_1(k,\omega)[y_1(k,\omega) - \imath\pi] = +V_2(k,\omega)[y_2(k,\omega) - \imath\pi]. \tag{122}$$

Therefore $y_1(k,\omega)$ is obtained as

$$\begin{aligned} y_1(k,\omega) &= \frac{1}{V_1^2(k,\omega) + V_1(k,\omega)V_2(k,\omega)}\{\frac{\omega^2 - \tilde{\omega}_{\mathbf{k}}^2}{\tilde{\omega}_{\mathbf{k}}} + \frac{1}{2}\int_{-\infty}^{\infty} d\omega' \frac{V_1(k,\omega)}{\omega' - \omega} \\ &+ \frac{1}{2}\int_{-\infty}^{\infty} d\omega' \frac{V_2(k,\omega)}{\omega' - \omega} - \imath\pi V_2^2(k,\omega)(\frac{V_1(k,\omega)}{V_2(k,\omega)} - 1)\}. \end{aligned} \tag{123}$$

Using (120) and (123), we find $\alpha_0(k,\omega)$ as

$$\alpha_0(k,\omega) = \frac{\omega + \tilde{\omega}_{\mathbf{k}}}{\sqrt{2}}(\frac{V_1(k,\omega) + V_2(k,\omega)}{\omega^2 - \tilde{\omega}_{\mathbf{k}}^2 + z_1(k,\omega) + z_2(k,\omega)}), \tag{124}$$

where $z_{\mathtt{i}}(k,\omega) \equiv \frac{\tilde{\omega}_{\mathbf{k}}}{2}\int_0^{\infty} \frac{V_{\mathtt{i}}^2(k,\omega)}{\omega - \omega' + i\epsilon}$. The other set of operators, $\hat{C}'(\mathbf{k},\omega)$, can be obtained from

$$V_1(k,\omega)[y_1'(k,\omega) - \imath\pi] = -V_2(k,\omega)[y_2'(k,\omega) - \imath\pi]. \tag{125}$$

For $\alpha_0'(k,\omega)$ we have

$$\alpha'_0(k,\omega) = \frac{\omega+\tilde{\omega}_{\mathbf{k}}}{\sqrt{2}}\left(\frac{V_1(k,\omega)-V_2(k,\omega)}{\omega^2-\tilde{\omega}^2_{\mathbf{k}}+z_1(k,\omega)+z_2(k,\omega)}\right). \qquad (126)$$

Eqs.(124) and (126) can be used to obtain C and C' in terms of $\hat{a}$, $\hat{a}^\dagger$, $\hat{B}_i$ and $\hat{B}_i^\dagger$. From (110), (119) and (121) we can write the Hamiltonian (107) as

$$\hat{H} = \int d^3\mathbf{k}\int_0^\infty \hbar\omega[\hat{C}^\dagger(\mathbf{k},\omega)\hat{C}(\mathbf{k},\omega)+\hat{C}'^\dagger(\mathbf{k},\omega)\hat{C}'(\mathbf{k},\omega)]. \qquad (127)$$

Using the commutation relation (119) and the commutation relation between $\hat{a}$ and $\hat{a}^\dagger$, we can invert the Eq.(119) to write $\hat{a}$ and $\hat{a}^\dagger$ in terms of $\hat{C}$, $\hat{C}^\dagger$, $\hat{C}'$ and $\hat{C}'^\dagger$ as

$$\begin{aligned}\hat{a}(\mathbf{k}) &= \int_0^\infty d\omega\{\alpha_0^*(k,\omega)\hat{C}(\mathbf{k},\omega)-\beta_0(k,\omega)\hat{C}^\dagger(\mathbf{k},\omega)\\ &+ \alpha_0'^*(k,\omega)\hat{C}'(\mathbf{k},\omega)-\beta'_0(k,\omega)\hat{C}'^\dagger(\mathbf{k},\omega)\}. \qquad (128)\end{aligned}$$

Before writing the EM field in terms of $\hat{C}$ and $\hat{C}^\dagger$, for later simplification we use a Bogoliubov transformation as

$$\hat{K}_e(\mathbf{k},\omega) = \frac{\hat{C}(\mathbf{k},\omega)+\hat{C}'(\mathbf{k},\omega)}{\sqrt{2}}, \qquad (129)$$

$$\hat{K}_m(\mathbf{k},\omega) = \frac{\hat{C}(\mathbf{k},\omega)-\hat{C}'(\mathbf{k},\omega)}{\sqrt{2}}. \qquad (130)$$

Using (59) and (128), $\hat{\mathbf{A}}$ can be obtained in terms of eigenoperators of the Hamiltonian, as

$$\begin{aligned}\hat{\mathbf{A}}(\mathbf{r},t) &= \frac{\imath}{(2\pi)^{\frac{3}{2}}}\sqrt{\frac{\hbar}{2\varepsilon_0}}\int d^3\mathbf{k}\int_0^\infty d\omega\{[\frac{\omega f_1(\omega)\hat{K}_e(\mathbf{k},\omega)}{\omega^2_{\mathbf{k}}(1-\chi_m(\omega))-\omega^2(1+\chi_e(\omega))}\\ &+ \frac{\omega_{\mathbf{k}} f_2(\omega)\hat{K}_m(\mathbf{k},\omega)}{\omega^2_{\mathbf{k}}(1-\chi_m(\omega))-\omega^2(1+\chi_e(\omega))}]e^{\imath(\mathbf{k}\cdot\mathbf{r}-\omega t)}-H.c.\},\end{aligned} \qquad (131)$$

where

$$\chi_e(\omega)\equiv\frac{1}{2}\int_{-\infty}^{+\infty}d\omega'\frac{f_1^2(\omega')}{\omega-\omega'-\imath\varepsilon} = \frac{1}{2}P\int_0^\infty d\omega'\frac{f_1^2(\omega')}{\omega-\omega'}+\frac{1}{2}\imath\pi|f_1(\omega)|^2, \quad (132)$$

and

$$\chi_m(\omega) \equiv \frac{1}{2}\int_{-\infty}^{+\infty} d\omega' \frac{f_2^2(\omega')}{\omega-\omega'-\imath\varepsilon} = \frac{1}{2}P\int_0^{\infty} d\omega' \frac{f_2^2(\omega')}{\omega-\omega'} + \frac{1}{2}\imath\pi|f_2(\omega)|^2, \quad (133)$$

and $f_1(\omega) \equiv \sqrt{\tilde{\omega}_0 c^2 k_c^2 \frac{\alpha_1 g_1(\omega)}{\omega}}$ and $f_2(\omega) \equiv \frac{\alpha_2 g_2(\omega)}{\sqrt{c^2\tilde{\omega}_0\varepsilon_0\rho}}$.

Now the process of calculating $\hat{\mathbf{M}}$ in the previous section can be repeated for $\hat{\mathbf{P}}$ and $\hat{\mathbf{M}}$ ($\hat{\mathbf{P}}$ and $\hat{\mathbf{M}}$ are defined in (76) and (77) respectively). We find

$$\hat{\mathbf{P}}(\mathbf{r},t) = \int_0^{\infty} d\omega\{[\varepsilon_0\chi_e(\omega)\hat{\mathbf{E}}(\mathbf{r},\omega) + \hat{\mathbf{P}}_N(\mathbf{r},\omega)]e^{-\imath\omega t} + H.c.\}, \quad (134)$$

and

$$\hat{\mathbf{M}}(\mathbf{r},t) = \int_0^{\infty} d\omega\{[\kappa_0\chi_m(\omega)\nabla\times\hat{\mathbf{A}}(\mathbf{r},\omega) + \hat{\mathbf{M}}_N(\mathbf{r},\omega)]e^{-\imath\omega t} + H.c.\}, \quad (135)$$

where $\hat{\mathbf{P}}_N(\mathbf{r},\omega)$ and $\hat{\mathbf{M}}_N(\mathbf{r},\omega)$ are

$$\hat{P}_{N\lambda}(\mathbf{r},\omega) = \int d^3\mathbf{k}\sqrt{2\hbar\varepsilon_0 Im\chi_e}\hat{K}_{e\lambda}(\mathbf{k},\omega)e^{\imath\mathbf{k}\cdot\mathbf{r}}, \quad (136)$$

$$\hat{M}_{N\lambda}(\mathbf{r},\omega) = \int d^3\mathbf{k}\sum_{\lambda'=1,2}\sqrt{2\hbar\varepsilon_0 c^2 Im\chi_m}\hat{K}_{m\lambda'}(\mathbf{k},\omega)e^{\imath\mathbf{k}\cdot\mathbf{r}}\epsilon_{\lambda\lambda'}. \quad (137)$$

By comparing (134), (135) and (8), (9) we find that χ_e and χ_m are electric and magnetic susceptibilities. As in the preview section, the commutation relation between $\hat{\mathbf{P}}_N(\mathbf{r},\omega)$ and $\hat{\mathbf{P}}_N^\dagger(\mathbf{r},\omega)$ and $\hat{\mathbf{M}}_N(\mathbf{r},\omega)$ and $\hat{\mathbf{M}}_N^\dagger(\mathbf{r},\omega)$ can be calculated. The results are compatible with the dissipation-fluctuation theorem and coincide with macroscopic results.

Using relation (131), (132) and Eq.(133), we find

$$\begin{aligned}\hat{\mathbf{A}}(\mathbf{r},t) &= -\imath(\frac{1}{8\pi^4\varepsilon_0})\int d^3\mathbf{k}\int_0^{\infty} d\omega\{[\frac{\omega\sqrt{Im\chi_e(\omega)}\hat{K}_e(\mathbf{k},\omega)}{\omega_{\mathbf{k}}^2(1-\chi_m(\omega))-\omega^2(1+\chi_e(\omega))}\\ &+ \frac{\omega_{\mathbf{k}}\sqrt{Im\chi_m(\omega)}\hat{K}_m(\mathbf{k},\omega)}{\omega_{\mathbf{k}}^2(1-\chi_m(\omega))-\omega^2(1+\chi_e(\omega))}]e^{-\imath(\omega t-\mathbf{k}\cdot\mathbf{r})} - H.c.\}.\end{aligned} \quad (138)$$

The relations (138), (132) and (133), and the commutation relations between the noise operators are exactly the same as the results obtained from the macroscopic method [12, 15]. So, these two methods are equivalent.

5 conclusion

The Huttner-Barnett model has been extended to a magnetodielectric medium. The results obtained in the present model are equivalent with those obtained in the phenomenological models. The explicit form of the noise operators have been obtained. Based on the results obtained here, the Lagrangian introduced in the present work can be used as a microscopic model for canonical quantization of the electromagnetic field in a magnetodielectric medium.

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